

# The optical behaviour of BL Lacertae at its maximum brightness levels: a blend of geometry and energetics

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## ABSTRACT

In 2021 BL Lacertae underwent an extraordinary activity phase, which was intensively followed by the Whole Earth Blazar Telescope (WEBT) Collaboration. We present the WEBT optical data in the *BVRI* bands acquired at 36 observatories around the world. In mid-2021 the source showed its historical maximum, with  $R = 11.14$ . The light curves display many episodes of intraday variability, whose amplitude increases with source brightness, in agreement with a geometrical interpretation of the long-term flux behaviour. This is also supported by the long-term spectral variability, with an almost achromatic trend with brightness. In contrast, short-term variations are found to be strongly chromatic and are ascribed to energetic processes in the jet. We also analyse the optical polarimetric behaviour, finding evidence of a strong correlation between the intrinsic fast variations in flux density and those in polarization degree, with a time delay of about 13 h. This suggests a common physical origin. The overall behaviour of the source can be interpreted as the result of two mechanisms: variability on time-scales greater than several days is likely produced by orientation effects, while either shock waves propagating in the jet, or magnetic reconnection, possibly induced by kink instabilities in the jet, can explain variability on shorter time-scales. The latter scenario could also account for the appearance of quasi-periodic oscillations, with periods from a few days to a few hours, during outbursts, when the jet is more closely aligned with our line of sight and the time-scales are shortened by relativistic effects.

**Key words:** galaxies: active – BL Lacertae objects: general – BL Lacertae objects: individual: BL Lacertae – galaxies: jets.

## 1 INTRODUCTION

Active galactic nuclei (AGNs) are among the most powerful sources in the Universe. Their central engine is a supermassive black hole

fed by an accretion disc. Some AGNs exhibit two plasma jets launched roughly perpendicularly to the accretion disc. In blazars, one relativistic jet is directed towards us, so that the jet emission undergoes Doppler boosting. This implies a series of effects, among which are an enhancement of the flux, a blueshift of the emitted frequencies, and a shortening of the variability time-scales (e.g. Urry & Padovani 1995; Blandford, Meier & Readhead 2019).

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The strength of these effects is described by the Doppler factor  $\delta = [\Gamma(1 - \beta \cos \theta)]^{-1}$ , where  $\Gamma = (1 - \beta^2)^{-1/2}$  is the bulk Lorentz factor,  $\beta$  is the plasma bulk velocity in units of the speed of light, and  $\theta$  is the viewing angle. Therefore, the relativistic effects become stronger if the plasma velocity increases or the orientation of the jet becomes closer to the line of sight.

Because of the Doppler boosting, the variable jet emission of blazars usually dominates over the other contributions, coming from the AGN (accretion disc and emission line regions) or host galaxy stars, and is usually observed at all wavelengths, from the radio to the  $\gamma$ -ray band. The study of blazar variability is a formidable tool to understand the structure of, and the physical mechanisms acting in, extragalactic jets.

The spectral energy distribution (SED) of blazars in the  $\log v_F$  versus  $\log v$  diagram shows two bumps, corresponding to low- and high-energy emission. The frequency at which these bumps peak varies from source to source and even for the same source at different epochs. The low-energy bump, extending from radio up to ultraviolet (UV) or even X-rays, is thought to be polarized synchrotron radiation produced by relativistic electrons in the magnetized plasma, whereas the high-energy bump, from X-rays to  $\gamma$ -rays, is likely predominantly due to inverse-Compton scattering of soft photons on the same relativistic electrons. However, the association between high-energy neutrinos of astrophysical origin revealed by neutrino detectors, and blazar jets as possible cosmic accelerators of these particles (e.g. IceCube Collaboration et al. 2018a,b; Giommi et al. 2020), makes it probable that also hadronic processes are involved in the production of high-energy photons (e.g. Böttcher et al. 2013).

Detailed analysis of blazar variability requires continuous monitoring. To increase the light-curve sampling, in particular in the optical band, the effort of many observers around the world is needed. This was the founding idea of the Whole Earth Blazar Telescope<sup>1</sup> (WEBT) Collaboration. In 25 yr of activity, the WEBT has analysed the multiwavelength behaviour of several blazars, proposing models to interpret it. One of the sources best studied by the WEBT is BL Lacertae (Villata et al. 2002, 2004a,b, 2009; Bach et al. 2006; Papadakis, Villata & Raiteri 2007; Raiteri et al. 2009, 2010, 2013; Larionov, Villata & Raiteri 2010; Weaver et al. 2020; Jorstad et al. 2022), the prototype of the BL Lac blazar class. In most of these previous works, we proposed that the long-term variability can be due to changes of the Doppler factor, most likely produced by variations in the orientation of the emitting regions with respect to the line of sight. In contrast, the short-term variability was ascribed to energetic processes occurring inside the jet. This scenario was found to be a viable explanation also for the variability observed in other blazars (see e.g. Raiteri et al. 2017b, 2021a,b). Moreover, in Jorstad et al. (2022) the excellent sampling reached by the WEBT allowed us to recognize a phase of quasi-periodic oscillations (QPOs) of the optical flux, optical polarization, and  $\gamma$ -ray flux. This transient phenomenon was explained as caused by the development of a current-driven kink instability in the jet, which was caused by the passage of an off-axis disturbance past a recollimation shock. These oscillations, with a period of about 13 h, were detected during the first, most dramatic phase of the 2020 outburst. In this new paper on BL Lacertae, we present optical photometric and polarimetric data with extremely dense sampling acquired by the WEBT during the subsequent 2021–2022 observing season, which was characterized by the source achieving historic brightness levels and displaying rapid variability on multiple time-scales. We aim to investigate

which are the persistent features in BL Lacertae optical variability behaviour, which features are instead transient, and what this can tell us about the source. A long-term analysis of the multiwavelength behaviour of the source from the radio to the  $\gamma$ -ray band will follow in a subsequent paper (Raiteri et al., in preparation).

This paper is organized as follows. The optical multiband photometric observations are presented in Section 2. In Section 3, the intraday variability (IDV) is analysed, while Section 4 deals with colour indices and spectral variability. A wavelet analysis of the flux density behaviour is performed in Section 5. The results of optical polarimetric monitoring are reported in Section 6. In Section 7, we discuss the twisting jet model that we have already successfully applied to explain blazar variability in previous papers, and in Section 8, we investigate the correlation between the flux and the degree of polarization. Conclusions are drawn in Section 9.

## 2 PHOTOMETRIC OBSERVATIONS

As mentioned in the Introduction, the WEBT Collaboration has been intensively monitoring the behaviour of BL Lacertae over more than two decades, and in particular, closely following the most recent activity phase that started in 2020. The WEBT  $R$ -band light curve from 2020 March 1 to December 31 was published by Jorstad et al. (2022), while in this paper we present data from 2021 January 1 to 2022 February 28. In this period we collected 24 765 data in four optical bands: 4642 in  $B$ , 5370 in  $V$ , 12 293 in  $R$ , and 2460 in  $I$ .

These new data were provided as 43 different data sets (see Table 1) from 36 observatories spread in longitude around the Northern hemisphere. Because the BL Lacertae observed emission is contaminated by the light of the stars in the host galaxy, the WEBT observers are invited to follow common prescriptions to perform the data reduction, in order to subtract the host contribution more easily, and get as homogeneous results as possible. These prescriptions include using an aperture radius of 8 arcsec to extract the source photometry, and an annulus of 10 and 16 arcsec radii for the background. Calibration of the source magnitude is performed with respect to Stars B, C, and H of the photometric sequence by Bertaud et al. (1969) in the  $B$  band, and by Fiorucci & Tosti (1996) in the  $V$ ,  $R$ , and  $I$  filters.

Because we aim to obtain accurate light curves on which a meaningful data analysis can be performed, we assembled the different data sets with extreme care, comparing the source behaviour in the four bands day by day. The density of the sampling allowed us to robustly determine whether some data sets presented offsets with respect to the bulk of the others, which can still occur notwithstanding the common reduction and calibration recipes. Such offsets were corrected by shifting the deviating data sets by their mean magnitude difference with respect to the contemporaneous data belonging to the data sets tracing the main trend. Moreover, we removed data points with large errors (more than 0.1 mag) and those that were clear outliers. The definition of an outlier is sampling dependent, being more stringent in well-sampled nights with well-defined intranight coverage with small errors, and less stringent in less-sampled nights with more scattered data. Finally, we reduced the data scatter by performing some binning of noisy data from the same data set in the same night. We underline that this light-curve processing is absolutely necessary if one wants to obtain a reliable data set that can be used for further analysis.

At the end of the procedure, we were left with 23 212 data points ( $\sim 94$  per cent of the original sample; 3553 in  $B$ , 5098 in  $V$ , 12 121 in  $R$ , and 2441 in  $I$ ). The density of data sampling can be appreciated

<sup>1</sup><https://www.oato.inaf.it/blazars/webt/>

**Table 1.** Details of the 43 optical data sets contributing to this paper.

Data set	Country	Diameter	Filters	$N_{\text{obs}}$	Symbol	Colour
Abastumani	Georgia	70	<i>R</i>	671	◊	Dark green
Abbey Ridge	Canada	35	<i>BVRI</i>	268	△	Orange
Aoyama Gakuin	Japan	35	<i>BVRI</i>	18	□	Cyan
ARIES	India	104	<i>BVRI</i>	19	□	Blue
ARIES	India	130	<i>BVRI</i>	27	□	Green
Athens <sup>a</sup>	Greece	40	<i>R</i>	141	◊	Cyan
Beli Brezi	Bulgaria	20	<i>VR</i>	152	*	Blue
Belogradchik <sup>b</sup>	Bulgaria	60	<i>BVRI</i>	191	+	Cyan
Burke-Gaffney	Canada	61	<i>VR</i>	660	△	Dark green
Catania (Arena)	Italy	20	<i>BVR</i>	40	×	Cyan
Catania (GAC)	Italy	25	<i>VRI</i>	61	△	Cyan
Connecticut	USA	51	<i>VR</i>	434	*	Grey
Crimean (AP7p)	Russia	70	<i>BVRI</i>	368	×	Magenta
Crimean (ST-7)	Russia	70	<i>BVRI</i>	432	+	Magenta
Crimean (ST-7; pol) <sup>b</sup>	Russia	70	<i>R</i>	535	×	Dark green
Crimean (ZTSh) <sup>c</sup>	Russia	260	<i>R</i>	223	△	Red
Felizzano	Italy	20	<i>R</i>	14	*	Magenta
GiaGa	Italy	36	<i>BVR</i>	62	*	Black
Haleakala (LCO)	USA	40	<i>VR</i>	50	+	Blue
Hans Haffner	Germany	50	<i>BVR</i>	1254	○	Red
Hypatia	Italy	25	<i>R</i>	827	◊	Red
Lowell (LDT)	USA	430	<i>VR</i>	18	○	Magenta
Lulin (SLT)	Taiwan	40	<i>R</i>	592	×	Violet
McDonald (LCO)	USA	40	<i>VR</i>	117	×	Blue
Montarrenti	Italy	53	<i>BVRI</i>	434	○	Dark green
Monte San Lorenzo	Italy	53	<i>R</i>	165	○	Green
Mt. Maidanak	Uzbekistan	60	<i>BVRI</i>	2961	◊	Green
Osaka Kyoiku	Japan	51	<i>BR</i>	569	□	Orange
Perkins <sup>b</sup>	USA	180	<i>BVRI</i>	824	○	Blue
Roque (NOT; e2v) <sup>b</sup>	Spain	256	<i>BVRI</i>	124	+	Green
Rozhen	Bulgaria	200	<i>BVRI</i>	51	□	Red
Rozhen	Bulgaria	50/70	<i>BVRI</i>	181	×	Orange
Seveso	Italy	30	<i>VR</i>	69	+	Violet
Siena	Italy	30	<i>VRI</i>	2367	◊	Blue
Skinakas	Greece	130	<i>BVRI</i>	1976	×	Black
St. Petersburg <sup>b</sup>	Russia	40	<i>BVRI</i>	347	+	Orange
Teide (IAC80)	Spain	80	<i>VR</i>	308	*	Green
Teide (LCO)	Spain	40	<i>VR</i>	76	+	Black
Tijarafe	Spain	40	<i>BR</i>	2946	*	Red
Vidojevica <sup>d</sup>	Serbia	140	<i>BVRI</i>	647	□	Black
Vidojevica <sup>d</sup>	Serbia	60	<i>BVRI</i>	37	△	Black
West Mountain	USA	91	<i>BVR</i>	2152	△	Magenta
Wild Boar	Italy	24	<i>VR</i>	28	△	Green

Note. ‘LCO’ refers to telescopes belonging to the Las Cumbres Observatory global telescope network.

<sup>a</sup>University of Athens Observatory (UOAO).

<sup>b</sup>Also polarimetry.

<sup>c</sup>Only polarimetry.

<sup>d</sup>Astronomical Station Vidojevica.

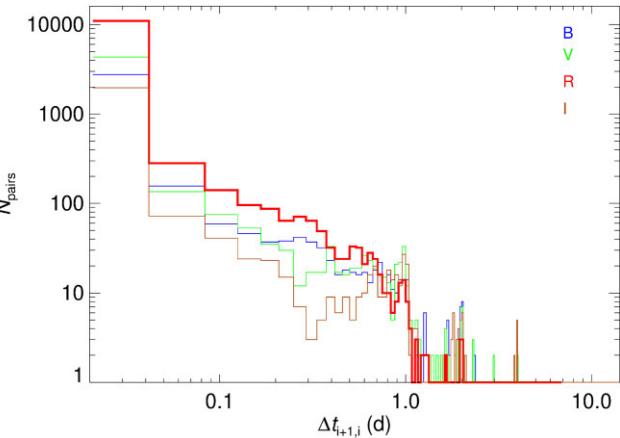
in Fig. 1, where the time difference between subsequent data points is plotted for all bands in bins of 1 h each. In the case of the best-sampled *R* bands, more than 90 per cent of data pairs are contained in the first bin.

The resulting cleaned light curves are shown in Fig. 2 in observed magnitudes. The peak of the mid-2021 outburst represents the observed historical maximum, with  $B = 12.75 \pm 0.02$ ,  $V = 11.75 \pm 0.02$ ,  $R = 11.14 \pm 0.03$ , and  $I = 10.47 \pm 0.01$  on August 6–7. The overall variation amplitude, defined as the maximum minus minimum magnitude, is 2.40, 2.36, 2.31, and 2.17 in the *B*, *V*, *R*, and *I* filters, respectively. Although the light curves have different sampling, the increasing amplitude variation with increasing frequency is typical of BL Lac objects. An enlargement of the *R*-band light

curve of Fig. 2 is shown in Fig. 3 to better appreciate the source short-term variability.

In the following analysis we will also use flux densities in mJy. They were obtained from the observed magnitudes using the zero-mag flux densities by Bessell, Castelli & Plez (1998), after correcting for the Galactic extinction values given by the NASA/IPAC Extragalactic Database<sup>2</sup> (NED): 1.192, 0.901, 0.713, and 0.495 mag in the *B*, *V*, *R*, and *I* bands, respectively. We also corrected for the emission contribution of the host galaxy according to Raiteri et al. (2009, 2010), i.e. we adopted a host galaxy flux density of 1.297,

<sup>2</sup><https://ned.ipac.caltech.edu/>



**Figure 1.** Distribution of the time difference between subsequent data points in the final, cleaned light curves. Blue, green, red, and brown histograms refer to the  $B$ ,  $V$ ,  $R$ , and  $I$  bands, respectively. More than 90 per cent of the  $R$ -band data pairs have a time separation of less than 1 h.

2.888, 4.229, and 5.903 mJy in the  $B$ ,  $V$ ,  $R$ , and  $I$  bands, respectively, and subtracted 60 per cent of this flux, because this is the amount of contamination for an aperture radius of 8 arcsec with background taken in an annulus of 10 and 16 arcsec radii, as recommended by the WEBT prescriptions.

### 3 INTRADAY VARIABILITY

BL Lacertae is one of the blazars that are known for their noticeable short-term variability and, in particular, for their intraday variability (IDV). Many IDV episodes have been reported in the WEBT studies on this source cited in the Introduction and in other works (e.g. Miller, Carini & Goodrich 1989; Meng et al. 2017; Fang et al. 2022; Imazawa et al. 2023).

In the period considered in this paper, further episodes of IDV were observed, particularly during the brightest phases. Interesting examples are shown in Figs 4 and 5, where not only the  $R$ -band, but also the  $B$ -band light curve is very well sampled. An additional example of the very closely spaced time-sampling obtained by the WEBT Collaboration in the  $R$  band is shown in Fig. 6. Here it is particularly evident that the participation of observers well distributed in longitude can lead to an almost continuous monitoring of the source, revealing the details of its short-term variability.

In Fig. 7, we show the number of days for which an observed maximum minus minimum  $R$ -band magnitude difference,  $\Delta R_{\text{IDV}}$ , was seen for all days having a number of observations greater than 10. In 73 per cent of the 241 resulting cases, the IDV amplitude ranges from 0.05 to 0.25 mag, but there are 51 d in which it is greater than 0.25 mag, with a maximum value of  $\sim 0.7$  mag.

These are of course lower limits, because they refer to what was observed, the actual total variability amplitude likely being larger.

When considering the IDV flux amplitude in the  $R$  band,  $\Delta F_{\text{IDV}}$ , as a function of the average flux density in the same band, we obtain the result plotted in Fig. 8, which suggests that the IDV amplitude increases with brightness. In the figure, we distinguish the cases where the number of intraday observations is  $< 30$  from those where it is  $\geq 30$ , and plot the corresponding linear fits to highlight that on average the amplitude increases with sampling. The different slope between the two fits stresses how the observed IDV amplitudes must be considered as lower limits to the total source IDV variations.

We note that larger flux variation in the brightest states is what one would expect if the long-term variability were due to changes of the Doppler factor (Raiteri et al. 2017b). This will be further discussed in the following sections.

### 4 SPECTRAL VARIABILITY

Blazars are known to exhibit specific spectral trends. In particular, BL Lac objects usually show a bluer-when-brighter behaviour. In the case of BL Lacertae, already Villata et al. (2002, 2004a) recognized that the long-term variability is almost achromatic, while the short-term variability shows a chromatic behaviour. Similar results have been found for other blazars, including S5 0716+714 (Raiteri et al. 2021a) and S4 0954+658 (Raiteri et al. 2021b).

Colour indices were obtained by first correcting the light curves for the host galaxy emission contribution and then coupling data points with errors smaller than 0.03 mag in the two filters, which were taken by the same telescope within 10 min. In this way we obtained 2629  $B - R$  colour indices, with values ranging from 1.42 to 1.79 and average  $\langle B - R \rangle = 1.61$  with standard deviation of 0.05 mag. In Fig. 9, we show the  $B - R$  colour indices plotted against both time and brightness, for the most well-sampled period corresponding to the major mid-2021 outburst.

In the bottom panel of Fig. 9, we also show the optical spectral index  $\alpha$  (assuming  $F_v \propto v^{-\alpha}$ ) derived from the colour index. Its values range from 1.53 to 2.43, and the mean value is  $\langle \alpha \rangle = 2.01$  with a standard deviation of 0.13. This implies that the optical SED is steep/soft, and therefore the peak of the synchrotron emission bump is located at lower frequencies, as expected in a low-energy-peaked BL Lac object (see Raiteri et al. 2009, 2010, 2013; Larionov et al. 2010).

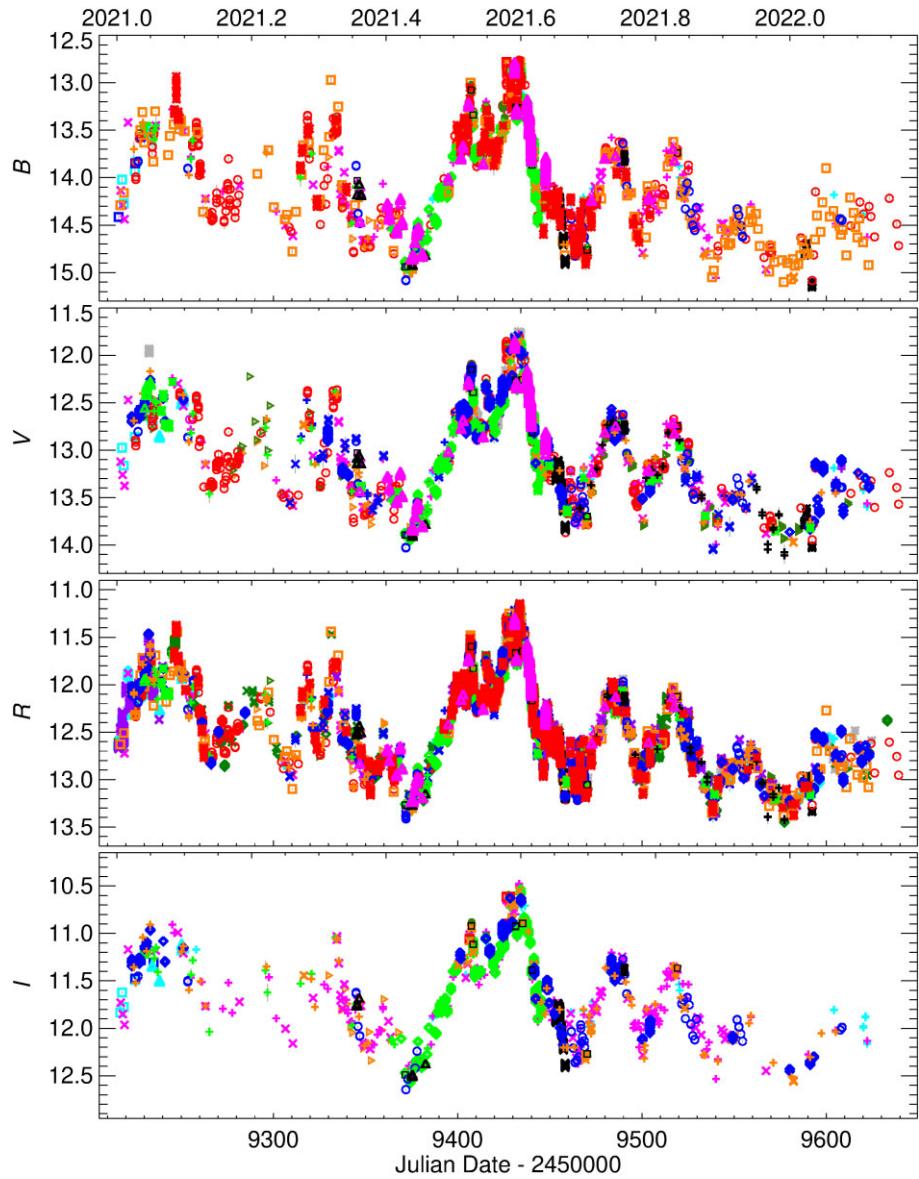
Furthermore, we identified, by visual inspection, examples of short-time intervals characterized by strong spectral variations; they are highlighted in Fig. 9. In these time intervals the source shows a clear bluer-when-brighter trend, with linear fit slopes ranging from 0.15 to 0.24. In contrast, a linear fit to the whole sample in the considered period indicates an almost achromatic behaviour, with a slope of only 0.04.

According to the interpretation given in the papers cited at the beginning of this section, the achromatic long-term trend would be due to Doppler factor variations of geometrical origin, while the chromatic short-term variability would be produced by intrinsic energetic processes.

### 5 VARIABILITY TIME-SCALES

Wavelet analysis is a powerful tool for detecting variations of power in time series, especially when they are not persistent, but occur only during defined time spans. The method is commonly adopted to unveil possible periodicities in blazar light curves at different frequencies and on a variety of time-scales (e.g. Gupta, Srivastava & Wiita 2009; Zhou et al. 2018; Gupta et al. 2019; Otero-Santos et al. 2020, 2023; Peñil et al. 2020; Jorstad et al. 2022; Roy et al. 2022).

To investigate the presence of characteristic variability time-scales in the BL Lacertae optical emission, we performed a wavelet analysis on the  $R$ -band flux density light curve. The results are shown in Fig. 10; they are based on the algorithm implementation by Torrence & Compo (1998). The wavelet power spectrum shows a series of significant periods, with confidence greater than 99 per cent, ranging from about 1 month down to a few hours. These periodic

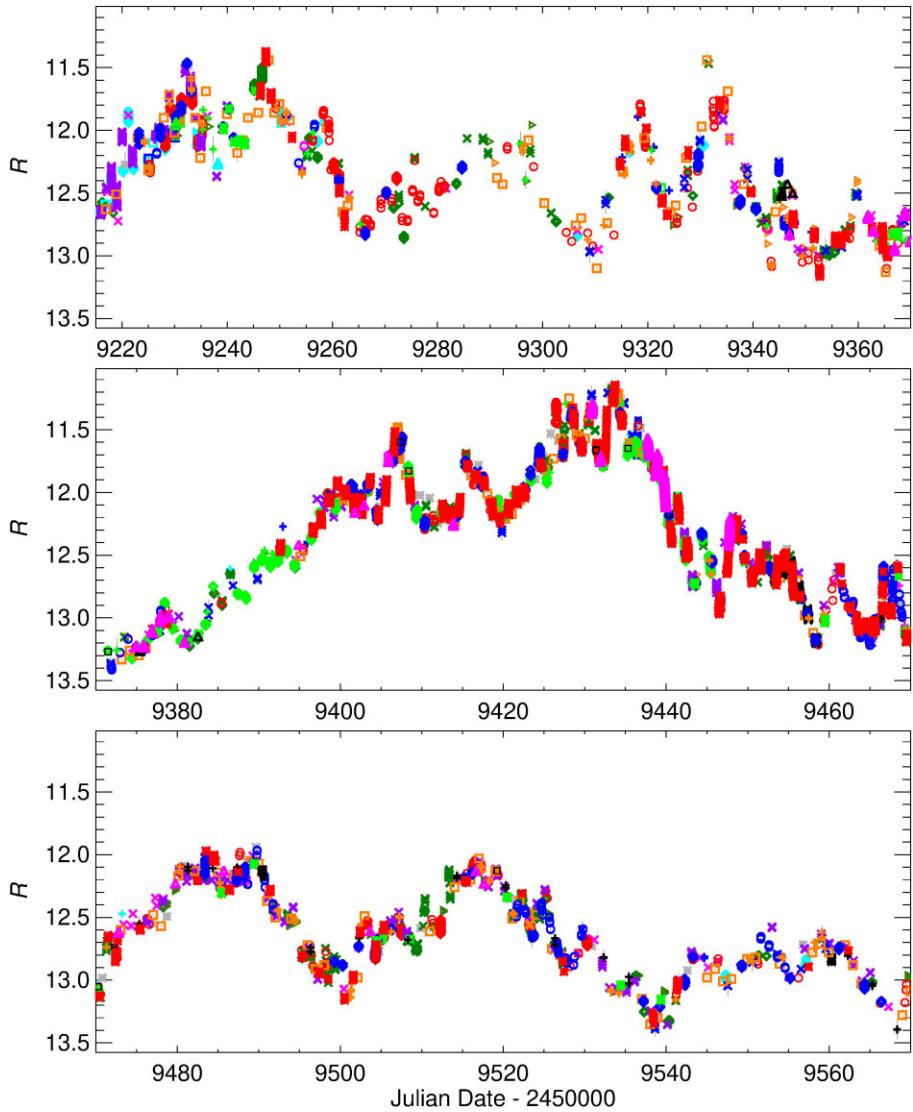


**Figure 2.** WEBT multiband optical light curves (observed magnitudes) of BL Lacertae during the 2021–2022 observing season. Different colours and symbols are used to distinguish the contributing data sets, as specified in Table 1. Measurement uncertainties are plotted in grey and are usually smaller than the data point size.

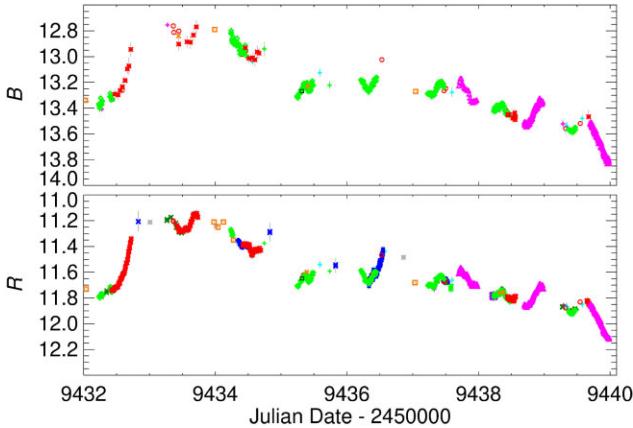
signals appear in correspondence of the three major flaring states, when the flux density exceeded about 100 mJy. In particular, the shortest scales (0.1–2 d) are more evident when the flux is higher. This is also highlighted by the plot of the variance, where the power is averaged over periods between 0.1 and 2 d. Indeed, the variance largely exceeds the 99 per cent confidence level in the brightest states. The global wavelet spectrum, which represents an average of the power in time over the whole period, shows that time-scales from about half a day to 4 d are particularly strong, i.e. well above the 99 per cent confidence level. These results are in agreement with those of the wavelet analysis performed by Jorstad et al. (2022), who found short-term QPOs of 0.55, 4, and 14 d during the 2020 outburst, and indicate that fast quasi-periodic flux oscillations are common in major flaring states of BL Lacertae.

## 6 OPTICAL POLARIMETRY

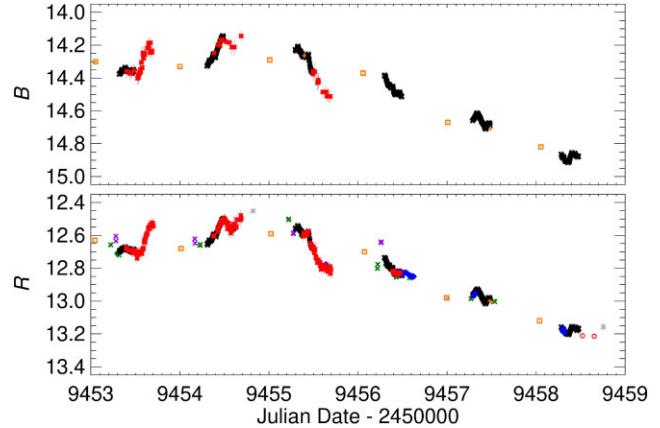
Interpreting the polarization variability in blazars has always been a challenge. Sometimes the optical flux and polarization degree,  $P$ , are observed to be correlated, sometimes they seem anticorrelated, and often they are uncorrelated (e.g. Raiteri et al. 2012, 2013; Rajput et al. 2022). Several studies concentrated on the wide rotations of the electric vector position angle (EVPA; see e.g. Larionov et al. 2008, 2016, 2020; Marscher et al. 2008, 2010; Sasada et al. 2012; Raiteri et al. 2013, 2017a; Blinov et al. 2015; Carnerero et al. 2015) that in some occasions appear correlated with  $\gamma$ -ray flares (Blinov et al. 2018) and may be produced by magnetic reconnection (Zhang et al. 2018). But some large rotations of EVPA can also be the result of stochastic processes, i.e. turbulence (e.g. Marscher 2014; Blinov et al. 2015; Kiehlmann et al. 2016, 2017; Raiteri et al. 2017a).



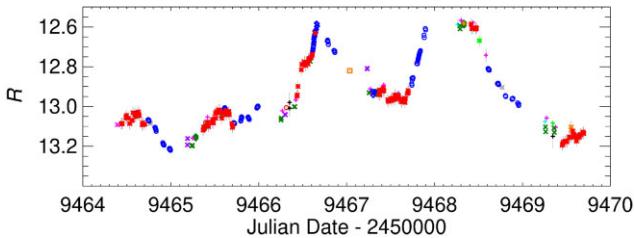
**Figure 3.** An enlargement of the  $R$ -band light curve shown in Fig. 2.



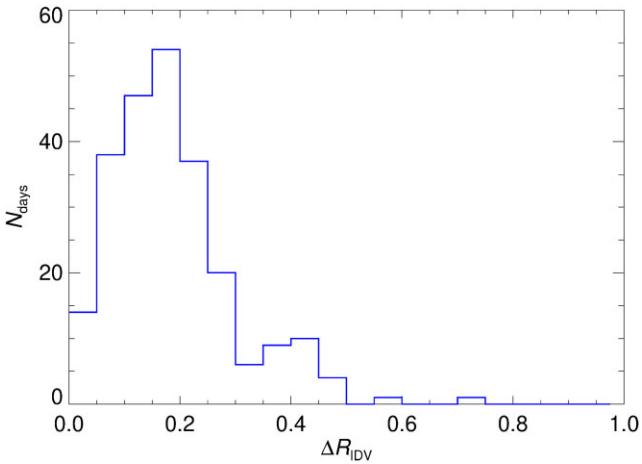
**Figure 4.** A detailed view of a portion of the  $B$  (top) and  $R$  (bottom) light curves covering the time period 2021 August 5–13, in which many IDV episodes can be seen thanks to the high-time resolution sampling. Measurement uncertainties are plotted in grey and are usually smaller than the data point size.



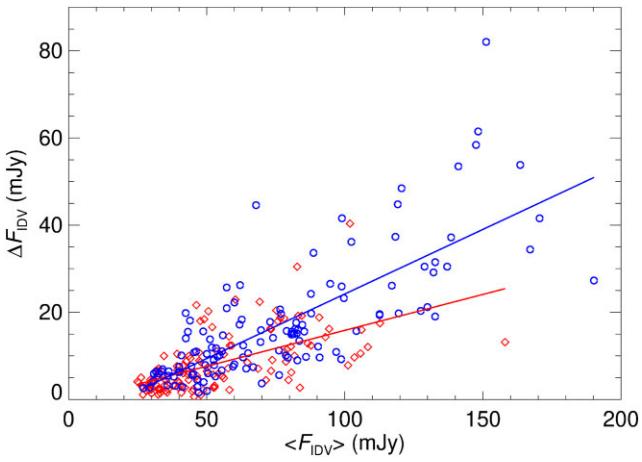
**Figure 5.** Same as Fig. 4, but for the period 2021 August 26–September 1.



**Figure 6.** A noticeable example of IDV in the  $R$  band in the period 2021 September 6–12.

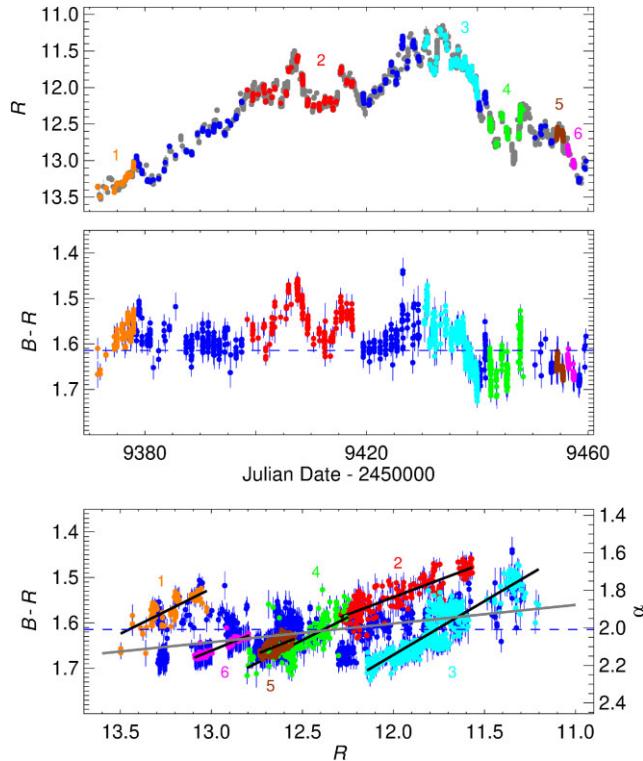


**Figure 7.** Number of days where the IDV magnitude amplitude in the  $R$  band is  $\Delta R_{\text{IDV}}$ , considering the 241 d where more than 10 observations were obtained in the same day.



**Figure 8.** IDV flux density amplitude in the  $R$  band as a function of the mean IDV flux density for the 241 d where more than 10 observations were obtained. Colours distinguish days with  $<30$  data points (red diamonds) from those with  $\geq 30$  observations (blue circles). Lines represent linear fits to the two samples.

During the 2021–2022 observing season, 1075 optical polarimetric data of BL Lacertae were acquired in the  $R$  band at the 60-cm telescope of the Belogradchik Observatory, at the 70- and 260-cm telescopes of the Crimean Observatory, at the 256-cm Nordic Optical Telescope (NOT), at the 180-cm Perkins Telescope, and at the 40-cm telescope of the St. Petersburg Observatory (see Table 1).

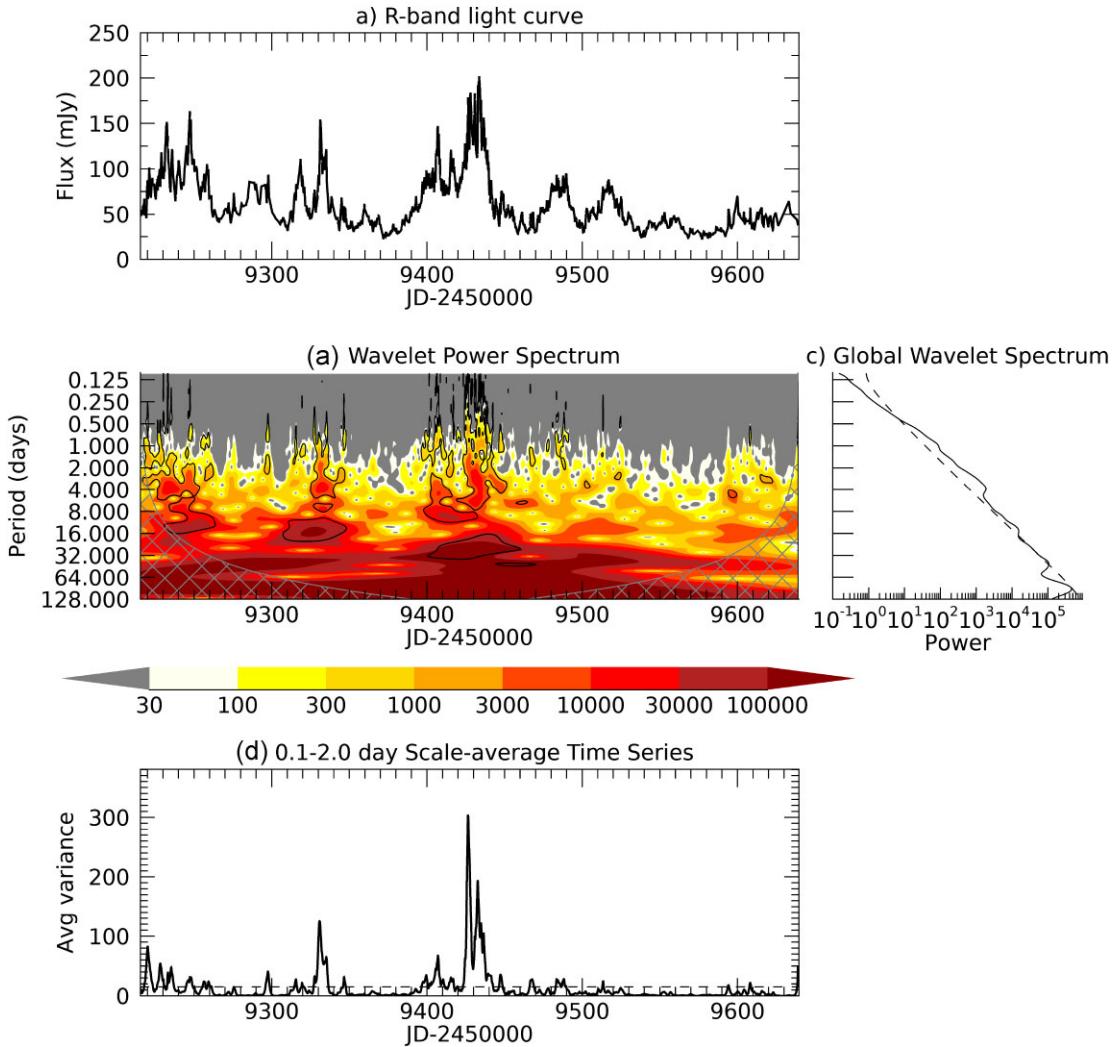


**Figure 9.** Top:  $R$ -band magnitudes during the main mid-2021 outburst, with the host galaxy emission contribution subtracted. Middle:  $B - R$  colour indices versus time. Bottom: the same colour indices versus brightness. Periods of strong chromatic variability are labelled by number and highlighted with different colours. In the bottom panel the black lines represent linear fits to the coloured data points, while the grey line refers to all the data shown in the figure.

In Fig. 11,  $P$  and the EVPA are shown as a function of time, and are compared to the behaviour of the optical flux density in the same band. The observed  $P$  ranges from 0.025 per cent to 25 per cent, with a mean value of 10 per cent and a standard deviation of  $\sim 4$  per cent. The dilution effect of the unpolarized light of the host galaxy (Raiteri & Villata 2021) has a negligible effect because of the source brightness.

The EVPA is known to present a  $\pm n \times 180^\circ$  ambiguity, which makes the reconstruction of its behaviour challenging, if the sampling presents interruptions. In the case of blazars, it has been shown that the sampling should be at least daily to obtain robust results (Kiehlmann et al. 2021). To handle this problem, we first constrained all EVPA values between  $0^\circ$  and  $180^\circ$ . Then, when the absolute value of the difference between the value of EVPA at a given time and its predecessor exceeded  $90^\circ$ , we added/subtracted  $180^\circ$  to minimize this difference. We note the following interesting features.

- (i) The degree of polarization  $P$  shows considerable IDV.
- (ii) On JD = 245 9345.55,  $P$  rapidly reaches the maximum value seen during our 2021–2022 observations. This does not correspond to a major event in flux, but rather to a minor peak.
- (iii) Starting from JD  $\sim 245 9500$ ,  $P$  shows an increasing trend of the baseline level, which is opposite to the trend of the optical flux density.
- (iv) The EVPA displays noticeable variability.
- (v) When the  $\pm n \times 180^\circ$  ambiguity is fixed as explained above, an extremely wide rotation of the EVPA ( $\gtrsim 360^\circ$ ) appears, starting from



**Figure 10.** Results of the wavelet analysis. (a)  $R$ -band flux densities of BL Lacertae in the 2021–2022 observing season. (b) Wavelet power spectrum: the strength of the power is colour coded according to the underlying palette; black contours define regions with confidence greater than 99 per cent; the grey grid represents the ‘cone of influence’ affected by edge effects. (c) Global wavelet spectrum with 99 per cent confidence level marked by a dashed line. (d) Time series of averaged periods between 0.1 and 2 d, with 99 per cent confidence level represented by the dashed line.

about the middle of the major 2021 outburst, i.e.  $JD \sim 245\,9430$ . This EVPA rotation proceeds in steps.

We note that the anticorrelation between the bulk behaviour of  $P$  and  $F_R$  was explained by Raiteri et al. (2013) in the framework of a geometrical scenario (where long-term flux variations are produced by changes of the viewing angle), by assuming a low value of the Lorentz factor. They also showed that the same geometric interpretation, but with a high Lorentz factor, can instead lead to correlation between flux and polarization degree.

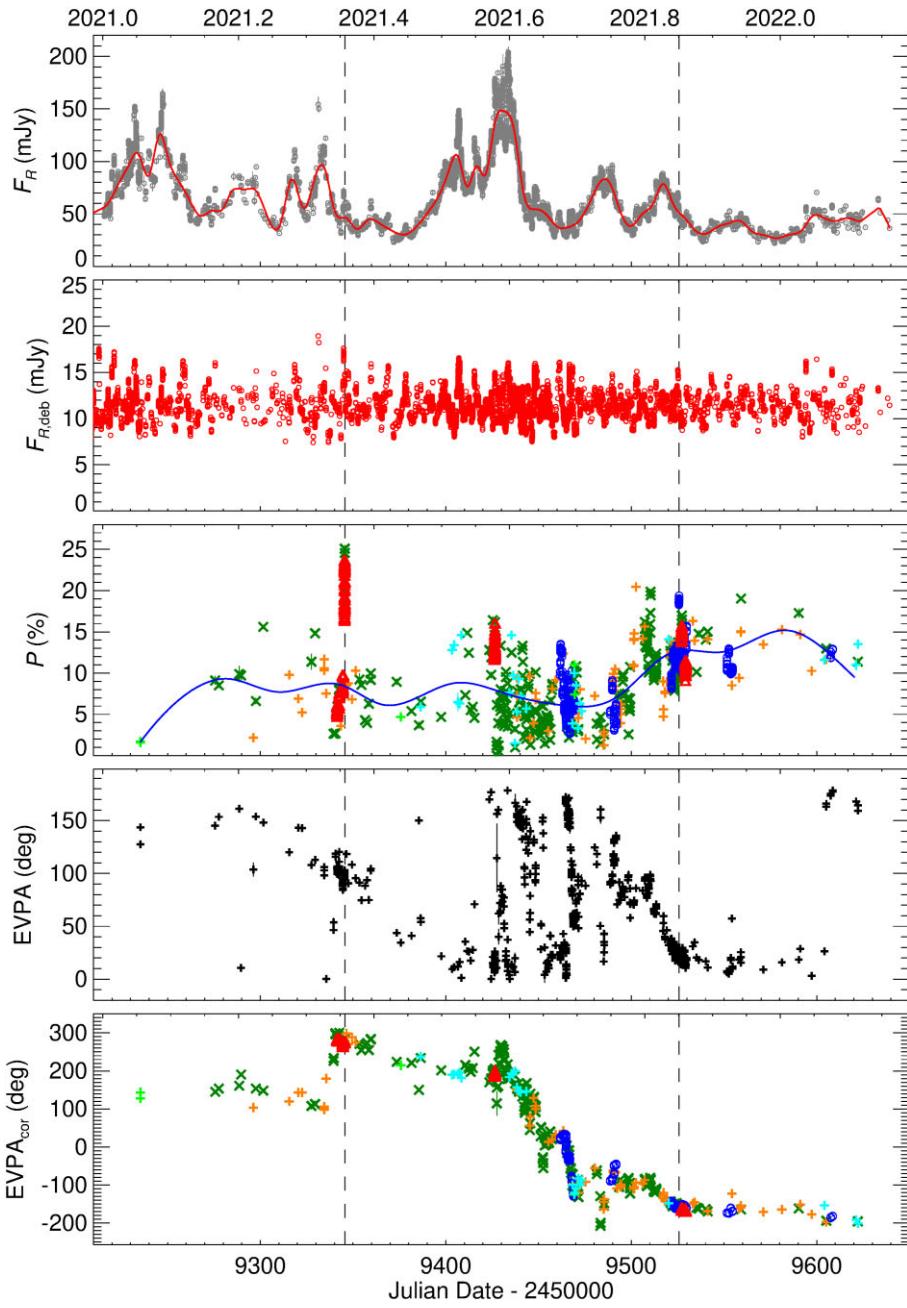
We also note that BL Lac sources generally show a preferred EVPA direction (Smith 1996), and in BL Lacertae this was found to be  $15^\circ$ – $25^\circ$  (Blinov & Hagen-Thorn 2009; Raiteri et al. 2013; Jorstad et al. 2022), indicating a magnetic field roughly perpendicular to the direction of the radio jet at 43 GHz (Jorstad et al. 2022). In the period of strong activity that we are considering, the EVPA shows intense variability. If we plot the distribution of the EVPA values (see Fig. 12), a concentration around  $15^\circ$ – $20^\circ$  appears, so it seems that the above preferred direction still dominates. However, this is mostly due to the sampling. Indeed, if we average the EVPA values over 6-h

bins, the importance of the  $15^\circ$ – $20^\circ$  peak is strongly reduced and the distribution of EVPA values appears more uniform overall.

We finally note that large jumpy rotations of the radio EVPA of 0727–115 were reported by Aller, Hodge & Aller (1981), who suggested a rotation or quasi-circular motion in the source emitting region. Moreover, a wide, step-like rotation of the optical EVPA observed in the BL Lac object S4 0716+714 was successfully reproduced by Larionov et al. (2013) assuming a shock wave propagating in a helical jet or along a helical path in the jet.

## 7 THE TWISTING JET

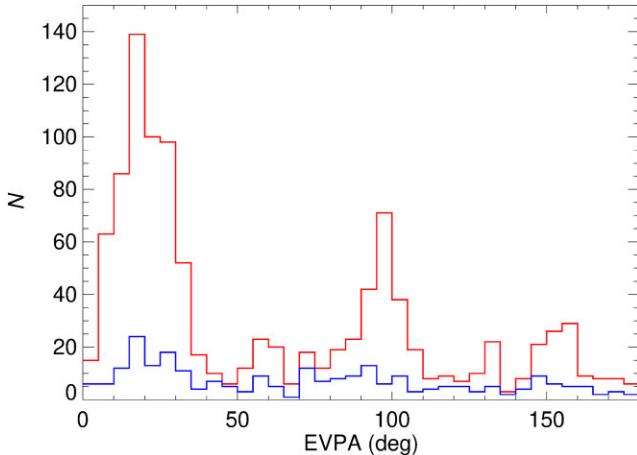
In previous papers, we have proposed that long-term variability in blazars has a geometrical origin, due to changes in the Doppler factor produced by variations in the orientation of the jet with respect to the line of sight (e.g. Villata et al. 2002, 2007; Raiteri et al. 2013, 2017b, 2021b). Indeed, the flux density at a given frequency  $\nu$  depends on the Doppler factor according to  $F_\nu \propto \delta^{n+\alpha}$ , where  $n = 2$  for a continuous jet (Urry & Padovani 1995) and  $\alpha$  is the optical spectral index (see Section 4). As mentioned in the Introduction, the Doppler



**Figure 11.** From top to bottom. (i) Optical flux densities in the  $R$  band; the red line represents the cubic spline interpolation on the binned light curve, with variable binning depending on brightness. (ii) Optical flux densities after deboosting as explained in Section 7. (iii) Observed optical polarization degree; the blue line represents a cubic spline interpolation through the 30-d binned data to highlight the long-term trend. (iv) EVPA constrained between  $0^\circ$  and  $180^\circ$ . (v) EVPA after arrangement for the  $\pm n \times 180^\circ$  ambiguity. In the third and last panels, different colours and symbols are used to distinguish the contributing data sets, as specified in Table 1. The vertical dashed lines mark the two events that are discussed in the text.

factor  $\delta$  depends on the viewing angle, so that variations in the jet orientation translate into changes in the source flux. Our model envisages an inhomogeneous and twisting jet. Inhomogeneous means that synchrotron radiation of increasing wavelength is emitted from jet regions that are at growing distance from the supermassive black hole, due to the interplay between emitted and absorbed frequencies, with opacity decreasing downstream the jet. This is suggested by the increasing time delay with which the radio flux variations at increasing wavelength follow those observed in the optical band in correlated optical–radio flares. Moreover, according to the model,

various emitting regions have different viewing angles, implying that the jet is curved, possibly helical. This comes from the fact that at a given epoch the brightness state can vary a lot with frequency, which is ascribed to different Doppler boosting. Finally, the orientation of the emitting regions changes over time, leading to a twisting jet. This scenario can be figured out by considering a rotating helical jet, where the minimum-viewing-angle (maximum Doppler boosting) zone shifts along the jet as the helix rotates. Starting, for example, with a minimum viewing angle in the optical-emitting region, we will observe an outburst in the optical band. But then the helix



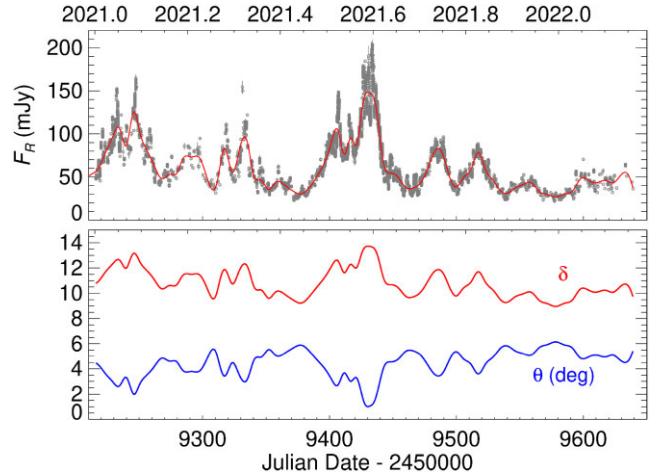
**Figure 12.** Distribution of EVPA values (red line). The major peak at  $15^\circ$ – $20^\circ$  corresponds to the preferential direction found by Raiteri et al. (2013) and Jorstad et al. (2022). The blue line shows the distribution of the mean EVPA values averaged over 6-h bins to check for the effect of sampling. Binning strongly reduces the dominance of the preferential direction and makes the distribution of EVPA values more uniform overall.

rotates, and the minimum viewing angle is progressively achieved by regions emitting longer and longer wavelength radiation, so that we will see time-delayed correlated events at these wavelengths. The most notable application of this model was the interpretation of the long-term variability behaviour of CTA 102 by Raiteri et al. (2017b).

It is worth mentioning that there may be several reasons why the jet is twisting, including orbital motion in a binary black hole system, jet precession, or plasma instabilities inside the jet. There is much evidence presented in many papers that support this interpretation, both observational (e.g. Perucho et al. 2012; Fromm et al. 2013; Casadio et al. 2015; Britzen et al. 2017, 2018; Issaoun et al. 2022; Zhao et al. 2022) and theoretical (e.g. Begelman, Blandford & Rees 1980; Nakamura, Uchida & Hirose 2001; Hardee 2003; Moll, Spruit & Obergaulinger 2008; Mignone et al. 2010; Liska et al. 2018).

In this paper, we found hints in favour of the above geometrical scenario for the long-term variability of BL Lacertae in 2021–2022. In Section 3, we saw that the variability amplitude increases with flux, and this is in agreement with the prediction that  $\Delta F_v \propto \delta^{n+\alpha}$ . Moreover, if the long-term trend is due to geometrical reasons, then the long-term spectral variability must be almost achromatic, as found in Section 4. Also the anticorrelation between the long-term trends of the flux and degree of polarization (Section 6 and 8) fits well in a geometric scenario, provided a relatively low Lorentz factor is assumed, as seems adequate for BL Lacertae (Jorstad et al. 2005; Raiteri et al. 2013). Finally, we found the appearance of short variability time-scales only during the source brightest states, which can be explained by the decrease in the time-scales due to Doppler beaming  $\Delta t = \Delta t' / \delta$ , where the prime refers to the rest frame. In conclusion, all these properties can be explained by assuming that the long-term behaviour of BL Lacertae is produced by variations of the Doppler factor of the jet-emitting regions due to changes in their orientation.

Therefore, we inferred the amount of variable beaming from the observed light curve, following the method explained in Raiteri et al. (2017b). Because relativistic beaming produces both a flux increase and a decrease of time-scales, we applied a variable binning to the  $R$ -band flux density light curve, where the time bin is reduced depending on the source brightness. Then we used a cubic spline



**Figure 13.** Top: optical flux densities in the  $R$  band; as in Fig. 11, the red line represents the cubic spline interpolation on the binned light curve, with variable binning depending on brightness, and is assumed to represent the long-term trend. Bottom: behaviour of the Doppler factor  $\delta$  and the viewing angle  $\theta$  in time derived from the long-term trend.

interpolation through the binned light curve to obtain a model for the long-term behaviour, which we assume to reflect the variations of the Doppler factor. This was then used to correct the flux densities for the effect of the relativistic beaming. The deboosted flux densities are plotted in Fig. 11 and show an almost constant variability amplitude. This further supports the validity of the Doppler changing scenario. We ascribe this residual short-term variability to physical energetic processes occurring inside the jet.

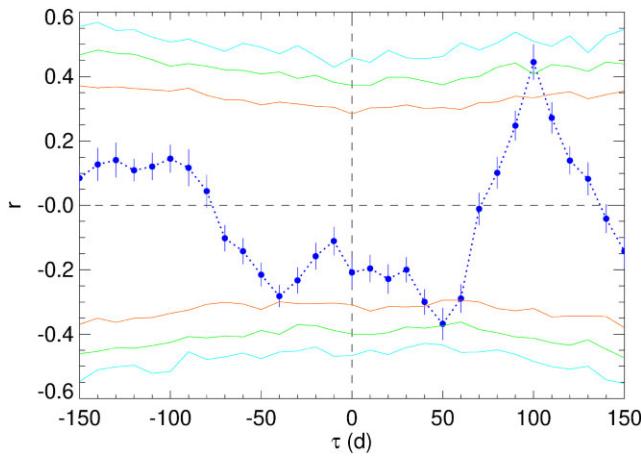
We finally note that in order to infer the behaviour of  $\delta(t)$  and  $\theta(t)$  from the long-term trend, we must assume a value for both the Lorentz factor and, for example, the minimum viewing angle. We adopted  $\Gamma = 7$  (Jorstad et al. 2005; Raiteri et al. 2013) and  $\theta_{\min} = 1^\circ$ , which implies  $\delta_{\max} \approx 13.73$ . Then  $\delta(t) = \delta_{\max} (F_{\text{spline}}(t) / F_{\text{spline,max}})^{1/(2+\alpha)}$  and  $\theta(t) = \arccos[(\Gamma \delta(t) - 1) / (\delta(t) \sqrt{\Gamma^2 - 1})]$ . The resulting  $\delta(t)$  and  $\theta(t)$  are shown in Fig. 13.

## 8 EXPLORING THE FLUX–POLARIZATION CORRELATIONS

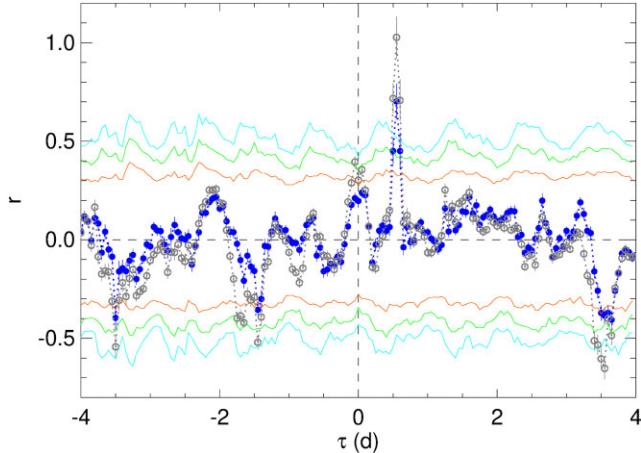
In Section 6, we noticed a general anticorrelation between the overall behaviour of the flux density and degree of polarization. We now want to investigate their relationship in more detail. To this end, we use the discrete correlation function (DCF; Edelson & Krolik 1988; Hufnagel & Bregman 1992), which is a method specifically conceived for unevenly sampled data trains. Our implementation of the algorithm performs a local normalization of the DCF, for more accurate results (Peterson 2001; Max-Moerbeck et al. 2014).

Fig. 14 shows the DCF between  $F_R$  and  $P$  for the whole period considered in this paper, adopting a data binning of 2 d and a DCF bin of 10 d. To estimate the significance of the DCF signals, we performed Monte Carlo simulations according to Emmanoulopoulos, McHardy & Papadakis (2013) and Max-Moerbeck et al. (2014). We produced 1000 artificial time series with the same power spectral density (PSD) and probability density function (PDF) properties as the flux densities, and other 1000 for the polarization degree, and then cross-correlated them. The resulting 80 per cent, 90 per cent, and 95 per cent confidence levels are displayed in Fig. 14.

The only noticeable peak, which, however, has a correlation coefficient  $r$  of only  $\sim 0.44$  and does not exceed the 95 per cent



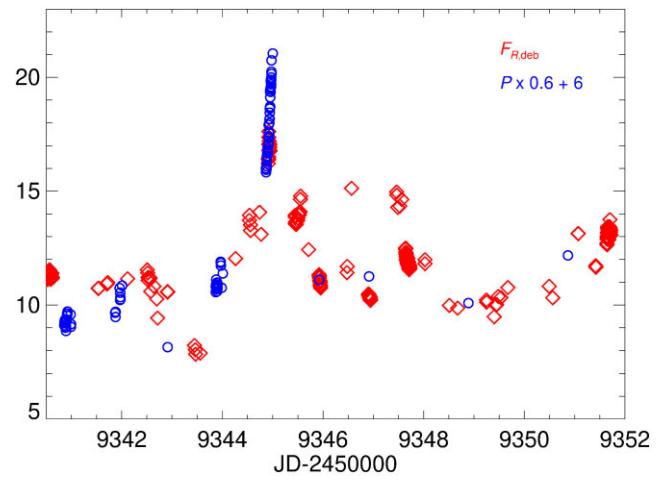
**Figure 14.** DCF between the  $R$ -band flux densities and the polarization degree, with data binning of 2 d and DCF binning of 10 d. The red, green, and cyan lines show the 80 per cent, 90 per cent, and 95 per cent confidence levels, respectively. These have been obtained through Monte Carlo simulations, as explained in the text.



**Figure 15.** DCF between the deboosted  $R$ -band flux densities and the polarization degree (blue dots). Data have been binned over 7.2 min intervals, and the DCF bin is 1.2 h. As in Fig. 14, the red, green, and cyan lines show the 80 per cent, 90 per cent, and 95 per cent confidence levels, respectively. The grey empty circles represent the DCF between the same deboosted  $R$ -band flux densities and the polarization degree after removal of the long-term trend, showing how detrending makes signals become stronger.

confidence level, suggests that  $P$  follows  $F_R$  with a delay of 100 d. This is the time interval between the flux major outburst phase around  $JD = 245\,9430$  and the high level of  $P$  reached about 100 d later and we do not attribute physical meaning to this signal. Instead, the anticorrelation at short time lags confirms what we have observed by visually inspecting Fig. 11. As already mentioned there, this anticorrelation can be explained in terms of orientation changes of the optical-emitting jet region given a low value of the Lorentz factor, as detailed in Raiteri et al. (2013).

We then analyse the correlation between the deboosted flux densities, i.e. what we consider the intrinsic jet flux variations, and  $P$  with much finer binning. The result of the DCF between the deboosted flux densities and the polarization degree is shown in Fig. 15. Data were preliminary binned over 7.2 min (0.005 d) and the DCF bin is 1.2 h (0.05 d). A noticeable peak whose value of



**Figure 16.** Deboosted  $R$ -band flux density (red diamonds) and polarization degree (blue circles) in two time intervals with good polarization sampling.  $P$  is rescaled as indicated in the legends and shifted in time by  $-0.55$  d.

the correlation coefficient is about 0.70 is found with a time delay of approximately 13 h. This signal gets enhanced if we remove the long-term trend from the polarization degree, dividing  $P$  by the cubic spline interpolation shown in Fig. 11. In this case the DCF peak has a value of 1.03. This means a strong correlation with a time delay of the polarization degree variations with respect to the intrinsic flux changes. We notice that the same time lag of about 0.55 d of the  $P$  variations after those of the flux density was also present in the past observing season according to the ZDCF analysis performed by Jorstad et al. (2022), although the correlation peak was lower. This indicates a persistent feature in the BL Lacertae behaviour.

To verify the robustness of the correlation signal, we estimated the significance of the DCF peak with the same method described above. The corresponding confidence levels confirm the significance of the correlation with time lag of 0.55 d.

This DCF signal actually comes from only two events, with  $P$  peaking at  $JD \sim 245\,9345.6$  (2021 May 11) and  $JD \sim 245\,9525.8$  (2021 November 7), which have good sampling in both flux density and polarization degree. They are shown in Fig. 16, where  $P$  is rescaled and shifted in time by  $-0.55$  d in order to better match the deboosted flux densities. The reasonably close overlap of  $F_R$  and  $P$  further indicates that the corresponding flares have roughly the same duration in time.

During these events, the source flux density was approximately at the same low level, and the EVPA remained almost constant at about  $101^\circ$  and  $19^\circ$ , respectively. This implies that the magnetic field in the first event was almost aligned with the direction of the jet as seen in the Very Long Baseline Array (VLBA) images at 43 GHz, while in the second event it was almost perpendicular (see Jorstad et al. 2022).

The strong correlation between the jet intrinsic flux variations (deboosted flux densities) and the variations of the polarization degree suggests that the same physical mechanism is responsible for both of them.

## 9 DISCUSSION AND CONCLUSIONS

In this paper, we have analysed the results of the optical photometric and polarimetric monitoring of BL Lacertae by the WEBT Collaboration in the 2021–2022 observing season. A very dense sampling was achieved thanks to the common observing effort of many observers using 41 telescopes around the Northern hemisphere. In this period, the source showed intense variability, with continuous intraday flux variations, and reached its historical brightness maximum. We have found evidence that the optical behaviour is well described by the twisting jet model, proposed in our earlier works to interpret the variability of BL Lacertae and other blazars. According to our interpretation, the long-term variability is due to geometrical reasons, i.e. to the variation of the orientation of the optical-emitting jet region with respect to the line of sight.

We have found several observational evidences supporting this view (see Section 7), including enhanced amplitude variability during outbursts, and an almost achromatic long-term spectral trend. In particular, by means of a wavelet analysis we have seen that significant variability short time-scales appear only when the source is in outburst. The shortest time-scales, from 2 d down to a few hours, are particularly evident in the brightest states. This is likely a consequence of the relativistic Doppler effect. Indeed, these brightest states would be due to a high Doppler factor, which would also shorten the intrinsic time-scales, leading to the appearance of the QPOs.

Moreover, also the anticorrelation between the long-term variations in flux densities and degree of polarization can be explained in the framework of our jet model, as discussed in Raiteri et al. (2013).

Once the flux densities are corrected for the effects of the variable Doppler boosting, we are left with fast flux changes of roughly the same amplitude. We recognize this short-term variability as the signature of energetic processes occurring inside the jet. In Section 8, we have found that variations in the deboosted flux densities are correlated with those in the degree of polarization, which are following with a 0.55 d delay. This correlation is based on a couple of well-sampled events, but was also present – though weaker – in the previous observing season (Jorstad et al. 2022), suggesting a common feature in the source behaviour. Interestingly, 0.55 d is also the period of the QPOs that Jorstad et al. (2022) detected in the optical flux and polarization, and in the  $\gamma$ -ray flux of BL Lacertae during the first phase of the 2020 outburst. In that case, the QPOs were explained as the result of a current-driven kink instability that was triggered by the passage of an off-axis perturbation through a stationary recollimation shock.

Magnetohydrodynamic simulations of relativistic plasma jets (RMHD) show that jets are subject to kink instabilities that twist the magnetic field lines and can disrupt the magnetic field structure. Magnetic reconnection then reorganizes the field, dissipating magnetic energy into particle energy (Begelman 1998; Sironi, Petropoulou &

Giannios 2015; Zhang et al. 2018, 2020, 2022; Dong, Zhang & Giannios 2020; Acharya, Borse & Vaidya 2021; Bodo, Tavecchio & Sironi 2021). The emission would come from plasmoids, dynamic plasma blobs/magnetic structures that appear in the reconnection region and that can have up to 10 per cent of its size (e.g. Petropoulou et al. 2018; Zhang et al. 2022). In simulations including polarization, flux peaks correspond to low  $P$  states, while  $P$  increases when the flux decreases, and the EVPA can either undergo large rotations or remain roughly constant. According to RMHD simulations by Dong et al. (2020), kink instabilities lead to a quasi-periodic release of energy. QPOs result also from the RMHD simulations by Acharya et al. (2021), who confirmed the validity of the twisting jet model. These kink-driven QPOs could explain the short time-scales appearing in the optical light curve of BL Lacertae during the brightest states (see Section 5). In this view, the QPOs time-scales could be linked to the dimension of the plasmoids, and the delay between the flux and  $P$  variations could represent the time needed to recover an ordered magnetic field, after its disruption by turbulence driven by the dissipation episode that energized the particles.

An alternative explanation for the correlation between  $F$  and  $P$  that we observed involves shock waves (e.g. Marscher & Gear 1985; Spada et al. 2001; Sironi et al. 2015; Böttcher & Baring 2019). Shocks could propagate through the optical-emitting region, promptly accelerating particles and gradually ordering the magnetic field. One caveat comes from the fact that during the first of the two events producing the correlation, the EVPA was about  $100^\circ$ , which means that the magnetic field was almost aligned with the radio jet. In contrast, during the second event the EVPA was about  $20^\circ$ , i.e. the magnetic field was approximately perpendicularly to the radio jet. If we assume a straight jet, it is difficult to imagine that these different situations can lead to the same lag between the variations in  $F$  and  $P$ . If we instead assume an inhomogeneous twisting jet, the jet direction in the optical-emitting region varies continuously as the jet twists, and it is likely different from the apparent orientation of the observed radio jet. The two events occurred in similar low-brightness states, which in our geometrical interpretation implies that the Doppler beaming was similar and low, i.e. in both cases the optical-emitting region had a similar viewing angle, but not necessarily the same orientation in space. Indeed, between the two events there is a complete rotation of the EVPA of more than  $360^\circ$ , which strengthens the idea of a twisting jet. In our view it is the jet that changes orientation, and we can have two different observed values of the EVPA in the optical-emitting region while the orientation of the magnetic field with respect to the jet direction remains constant. This could explain why shocks can produce similar events with different observed EVPAs.

A final remark is due regarding the consequences of the relativistic effects on the search for QPOs, since changes in the Doppler factor affect not only the source brightness, but also the time-scales of the intrinsic, energetic variations. If a typical time-scale exists in the jet rest frame, it will appear shorter in brighter states than in fainter ones. Therefore, the search of QPOs is meaningful only when performed during fairly constant brightness levels, as e.g. done in Jorstad et al. (2022). Otherwise, one must process the light curves in order to correct both the fluxes and the time intervals for the same Doppler factor changes. This argument can also be applied to the time delay of  $P$  versus  $F$ , and indeed we found the same delay of 0.55 d in similar brightness levels.

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## DATA AVAILABILITY

Data acquired by the WEBT Collaboration are stored in the WEBT archive and are available upon request to the WEBT President Massimo Villata (massimo.villata@inaf.it).

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